

# Contents

Preface . . . . .	i
<b>1 Topics in algebraic graph theory</b>	<b>1</b>
1 Graph spectrum . . . . .	2
1.1 Matrices associated to a graph . . . . .	2
1.2 The spectrum of a graph . . . . .	2
1.3 The spectrum of an undirected graph . . . . .	4
1.4 Connectedness . . . . .	5
1.5 Spectrum of some simple graphs . . . . .	6
1.6 Decompositions . . . . .	8
1.7 Automorphisms . . . . .	9
1.8 Exercises . . . . .	10
2 Linear algebra . . . . .	10
2.1 Hermitean linear transformations . . . . .	10
2.2 Perron-Frobenius Theory . . . . .	11
2.3 Equitable partitions . . . . .	14
2.4 Interlacing . . . . .	15
2.5 Exercises . . . . .	17
3 Application to adjacency matrices . . . . .	17
3.1 The largest eigenvalue . . . . .	17
3.2 Graphs with largest eigenvalue at most 2 . . . . .	18
3.3 Regular graphs . . . . .	19
3.4 Bipartite graphs . . . . .	20
3.5 The second largest eigenvalue of a regular graph . . . . .	21
3.6 Cliques and cocliques . . . . .	22
3.7 Chromatic number . . . . .	22
3.8 Exercises . . . . .	24
4 More applications of interlacing . . . . .	24
4.1 The Laplace matrix . . . . .	24
4.2 Block Designs . . . . .	27

4.3	Exercises . . . . .	30
5	Strongly regular graphs . . . . .	30
5.1	General results . . . . .	30
5.2	Strongly regular graphs with eigenvalue $-2$ . . . . .	33
5.3	Connectedness . . . . .	34
5.4	Cocliques and colorings . . . . .	35
5.5	Generalized quadrangles . . . . .	38
5.6	Other algebraic invariants . . . . .	41
5.7	Exercises . . . . .	42
6	Association schemes . . . . .	42
6.1	Definition . . . . .	42
6.2	The Bose-Mesner algebra . . . . .	44
6.3	The Linear Programming Bound . . . . .	46
6.4	The Krein parameters . . . . .	47
6.5	$P$ - and $Q$ -polynomial association schemes . . . . .	49
6.6	Distance regular graphs . . . . .	51
6.7	Exercises . . . . .	55
7	The $(81,20,1,6)$ strongly regular graph . . . . .	56
7.1	Descriptions . . . . .	56
7.2	Uniqueness . . . . .	57
7.3	Independence and Chromatic numbers . . . . .	59
7.4	Exercises . . . . .	60

<b>2</b>	<b>Combinatorial problems on Cayley graphs</b>	<b>67</b>
1	Introduction . . . . .	67
2	Definitions, examples and basic properties . . . . .	68
2.1	Groups and graphs . . . . .	68
2.2	Symmetry and regularity of graphs . . . . .	71
2.3	Examples . . . . .	73
2.4	Exercises . . . . .	80
3	Problems on Cayley graphs . . . . .	81
3.1	Hamiltonicity . . . . .	81
3.2	The diameter problem . . . . .	84
3.3	The vertex reconstruction problem . . . . .	87
3.4	Exercises . . . . .	91
4	Cayley graphs on the group $Sym_n$ . . . . .	92
4.1	The transposition graph $Sym_n(T)$ . . . . .	92
4.2	The bubble-sort graph $Sym_n(t)$ . . . . .	95
4.3	The star graph $Sym_n(st)$ . . . . .	96

4.4	The reversal graph $Sym_n(R)$	97
4.5	The pancake graph $Sym_n(PR)$	112
4.6	Exercises	114
5	Cayley graphs on the group $B_n$	115
5.1	The transposition graph $B_n(T^\sigma)$	115
5.2	The bubble-sort graph $B_n(t^\sigma)$	117
5.3	The star graph $B_n(st^\sigma)$	118
5.4	Some other cases	119
5.5	The reversal graph $B_n(R^\sigma)$	120
5.6	The burnt pancake graph $B_n(PR^\sigma)$	134
5.7	Exercises	135
6	Answers to exercises	136
7	Further reading	140
<b>3</b>	<b>Some applications of polynomials in combinatorics</b>	<b>145</b>
1	Two-distance sets	146
2	Polynomials restricted to rectangular subsets	147
3	Codes and set-systems with restricted distances	148
4	Applications of Theorem 6	152
5	The number of solutions of polynomial congruences	154
6	The $p$ -rank of desarguesian projective planes	158